Reply to Reviewer 2

The remarks and comments have been scrutinized. Among them, the challenging question of exact equivalence to the Planck constant has been fully answered in a positive manner. The rebuttal is organized by replying to each criticism in a detailed manner.

Reviewer 2 writes:

A close inspection of the proof shows that it is nothing more than a mathematical analogy between the (n +1/2) factor appearing in the harmonic oscillator quantized energy levels and the fact that there are 2n +1 = 2(n + 1/2) values of the m value in the n'th spherical harmonic. This is shown by the key equation (18) which involves the orthogonality of the spherical harmonics followed by a sum over the number of m values for fixed n

Author's reply:

The mathematical analogy cannot be discarded as it was without importance. It is the first time that this analogy with the energy levels of a quantum harmonic oscillator appears in the classical context of electrodynamics. This is undeniable and has a deep impact. As explained in the introduction, it is common knowledge that discrete energy levels cannot be obtained within the classical theory of light. This belief is disproved in the submitted manuscript.

Reviewer 2 writes:

That the n values in the spherical harmonics are integers has nothing to do with quantization.

Author's reply:

I kindly disagree with the Reviewer. The reasons of my disagreement deserve some explaining. First of all, conventional quantization is introduced by using the correspondence to the quantum harmonic oscillator [section 10.3, "Canonical quantization of the transverse field", pages 473-475 in L. Mandel, E. Wolf, Optical Coherence and Quantum Optics, Cambridge University Press, Cambridge, (1995)]. Secondly, the quantum harmonic oscillator has eigenvalues that are integers and correspond to eigenstates that are made of number states or Fock states. Given this premise, if the energy is countable (<u>thanks to the integer values of n</u>), then the quantization is well established. This is the difference with classical physics where energy is continuous or not countable. In the submitted work, the integer values of n are reached thanks to the spherical harmonics. The energy becomes discrete and countable. All the conditions of quantization are satisfied even though we use a classical route toward quantization. On the other hand, we can listen to Feynman who commented on the meaning of quantization as reported in his Lectures: "*There is no way to make up your mind whether the electromagnetic field is really to be described as a quantized harmonic oscillator or by giving how many photons there are in each condition. <u>The two views turn out to be mathematically identical</u>. So in the future we can speak either about the number of photons in a particular state in a box or the number of energy level associated with a particular mode of oscillation of the electromagnetic field*" (page 4-9, section 4-5, The Feynman Lectures on Physics, Vol. 3).

Reviewer 2 writes:

The partial wave expansion of a plane wave is as meaningful in classical physics as in quantum mechanics

Author's reply:

I agree on this comment on plane waves. However, in the multipole approach of this manuscript or in the multipole approaches found in various branches of physics, plane waves are not used and thus the Reviewer's comment does not apply. Nevertheless, in trying to find what is at stake here, where classical and quantum physics are compared, it is a fact that the submitted work shows the first proof that classical electromagnetism has the potential for recovering results that are believed to belong to quantum optics only. It means that, given an identical meaning of whatever wave expansion is used (either plane waves of the Reviewer's comment or spherical waves of the submitted work), it is of some relevance that classical physics can gain some ground on quantum physics.

Reviewer 2 writes:

If the author believes the proof then it should be shown that the constant <beta> is numerically equal to the known value of Planck's constant.

Author's reply:

The proof of equivalence between the constant <beta> and the Planck's constant has been added to the manuscript. In reality, this proof is redundant. This can be shown by means of a simple reasoning. If we accept the correspondence principle for Eq. (8), where the equivalence between the classical electromagnetic energy and the quantum harmonic oscillator is purposely designed to obtain the intended result of the conventional quantization, we should accept the same correspondence principle that makes Eq. (24) equal to Eq. (1). We cannot apply double standard in dealing with the classical and quantum energies of section 2 (conventional procedure of quantization) and the same energies of sections 3 and 4 (classical quantization based on the multipole approach). Therefore, if we have two theories where one says that

quantum energy = a * x

and the other finds

classical energy = b * x

we can conclude that a=b from the equivalence between the two energies. Compare Eq. (1) with Eq. (24) and the equivalence between the constant <beta> and the Planck's constant will come out in a simple manner. However, an independent proof has been given (new section 6 of the revised manuscript). It consists in the demonstration that the constant <beta> can be derived from a typical experimental constant of the blackbody (the Stefan-Boltzmann constant).