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Beyond quantum-classical analogies: high time for agreement?

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Outline

Brief introduction to quantum-classical analogs

Focus on the analogy between Lorenz-Mie (LM or Mie) scattering
and quantum-mechanical (qm) wave scattering

Application to the blackbody problem
(can we conceive a classical route to the blackbody?)

Conclusions

Quantum physics emerges from classical thinking

(1900 blackbody, 1912 vacuum field)

Planck borrowed from Boltzmann the idea about classical discretization of volume and phase space to calculate thermodynamic functions (e.g., energy, entropy)

$$n_i = \alpha g_i e^{-\beta E_i} = \underbrace{\frac{N}{Z} g_i e^{-\beta E_i}}_{\text{Maxwell - Boltzmann distribution law}} \qquad \underbrace{Z = \sum_i g_i e^{-\beta E_i}}_{\text{parition function:}} \longrightarrow \quad \langle n \rangle = \frac{1}{e^{\hbar\omega/k_B T} - 1}$$

Thermal distribution

(1925 matrix mechanics)

Born, Heisenberg and Jordan were inspired by the connection to the vibrating string

$$H = \frac{1}{2}\int_0^l \left\{u^2 + \left(\frac{\partial u}{\partial x}\right)^2\right\} \mathrm{d}x = \frac{l}{4}\sum_{k=1}^{\infty}\left\{\dot{q}_k(t)^2 + \left(k\frac{\pi}{l}\right)^2 q_k(t)^2\right\}.$$

Eq. (42) in the 1925 paper

Quantum harmonic oscillator (after quantization procedure)

(1926 wave mechanics)

Schrödinger worked on the optical analogy to achieve an interpretation of the de Broglie postulate about matter waves

Maxwell eq. of a free em field

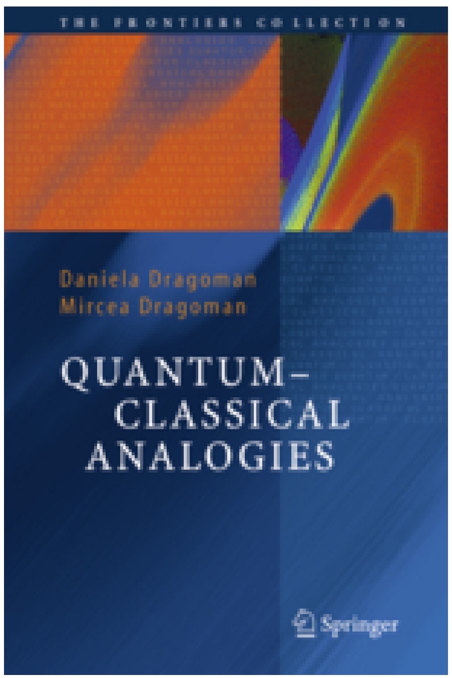
$$\nabla^2 A + k^2 A = 0$$

Schrödinger eq. of a massive particle

$$\nabla^2 \psi + \left[2m(E - V)/\hbar^2\right]\psi = 0$$

Quantum physics in classical fashion

A lot of papers have been written on quantum-classical analogies



Springer (2004)

PRL 102, 243601 (2009) PHYSICAL REVIEW LETTERS week ending 19 JUNE 2009

Classical Analogues of Two-Photon Quantum Interference

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Chirped-pulse interferometry (CPI) captures the metrological advantages of quantum Hong-Ou-Mandel (HOM) interferometry in a completely classical system. Modified HOM interferometers are the basis for a number of seminal quantum-interference effects. Here, the corresponding modifications to CPI allow for the first observation of classical analogues to the HOM peak and quantum beating. They also allow a new classical technique for generating phase super-resolution exhibiting a coherence length dramatically longer than that of the laser light, analogous to increased two-photon coherence lengths in entangled states.

PRL 107, 103601 (2011) PHYSICAL REVIEW LETTERS week ending 2 SEPTEMBER 2011

Classical Analogue of Displaced Fock States and Quantum Correlations in Glauber-Fock Photonic Lattices

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Coherent states and their generalizations, displaced Fock states, are of fundamental importance to quantum optics. Here we present a direct observation of a classical analogue for the emergence of these states from the eigenstates of the harmonic oscillator. To this end, the light propagation in a Glauber-Fock waveguide lattice serves as equivalent for the displacement of Fock states in phase space. Theoretical calculations and analogue classical experiments show that the square-root distribution of the coupling parameter in such lattices supports a new family of intriguing quantum correlations not encountered in uniform arrays. Because of the broken shift invariance of the lattice, these correlations strongly depend on the transverse position. Consequently, quantum random walks with this extra degree of freedom may be realized in Glauber-Fock lattices.

Lorenz-Mie scattering and qm wave scattering

The analogy has been investigated by prof. Gouesbet (Univ. of Rouen, France) in a number of publications

Optics Communications 231 (2004) 9–15

Cross-sections in Lorenz–Mie theory and quantum scattering: formal analogies

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Abstract

We consider the scattering of a plane wave by a sphere (Lorenz–Mie theory) and the corresponding quantum problem of scattering of an illuminating plane wave beam by a radial potential $U(r)$, and demonstrate that cross-sections in both frameworks are amenable to identical expressions, excepted for zero-order phase shift terms which are specific of the quantum framework.

Optics Communications 266 (2006) 710–715

A transparent macroscopic sphere is cross-sectionally equivalent to a superposition of two quantum-like radial potentials

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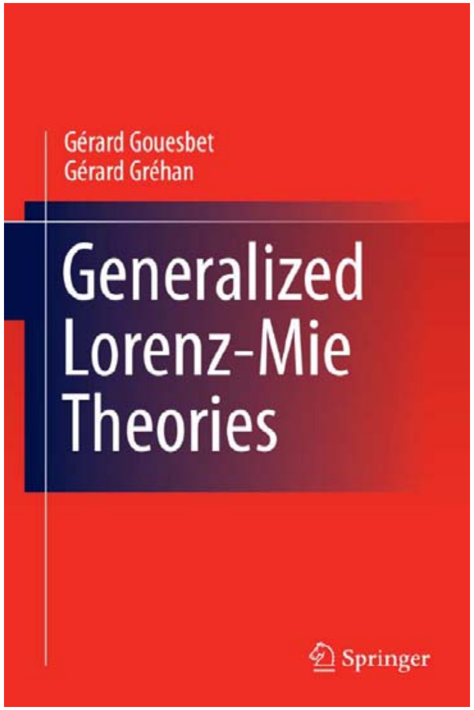
Abstract

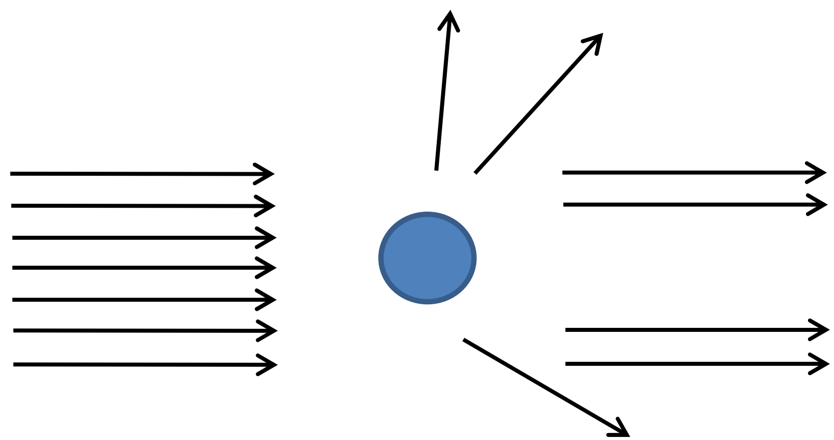
We consider two frameworks (i) the electromagnetic generalized Lorenz–Mie theory describing the interaction between an electromagnetic arbitrary shaped beam and a homogeneous, non-magnetic sphere, with an isotropic, linear, material and (ii) a quantum generalized Lorenz–Mie theory describing the interaction between a quantum eigen-arbitrary shaped beam and a quantum radial potential. For the time being, we restrict ourselves in this paper to elastic scattering cross-sections. We then demonstrate that a transparent macroscopic sphere in the first framework is equivalent to a superposition of two quantum-like radial potentials in the second framework. The restrictive meaning of "quantum-like" will be discussed when appropriate.



Springer (2011)

Mie or Lorenz-Mie theory about em scattering

Electromagnetic (em) plane waves scattered by spheres



(years 1880-1910)

Lorentz, Rayleigh, Mie, Debye, Thomson, Lorenz and many others

Mie or Lorenz-Mie theory

Kerker, “The scattering of light”, Academic Press (1969)
Bohren, Huffman “Absorption and Scattering of Light by small particles”, Wiley (1983)
Jackson, “Classical electrodynamics”, Wiley (1998)

Lorenz-Mie and quantum scattering: formal analogies

Classical cross-section

$$\sigma_{cl} = \frac{2\pi}{k^2}\sum_{n=1}^{\infty}(2n+1)\left(|a_n|^2+|b_n|^2\right)$$

$$kd << 1$$

$$|a_n|^2+|b_n|^2=|s_n|^2 \cong |s_0|^2 e^{-nq}$$

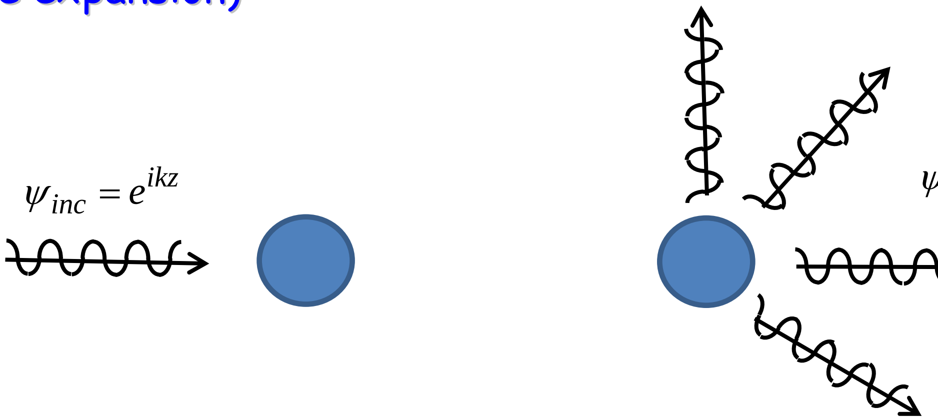
Note:

(1) under the approximation of small scatteres, the scattering coefficients decay exponentially,
(2) dependence on the odd order (2n+1),
(3) dependence on the inverse of k^2.

Quantum-mechanical cross-section (partial wave expansion)

$$\sigma_{qm} = \frac{4\pi}{k^2}\sum_{n=0}^{\infty}(2n+1)|s_n|^2$$

$$s_n = j_n(kd)/h_n^{(1)}(kd)$$



$$\psi_{inc} = e^{ikz}$$

$$\psi_{tot} = e^{ikz} + f(\theta)\frac{e^{ikr}}{r}$$

Quantitative analogy for elastic scattering

Opt Comm 231, 9-15 (2004)

Cross-sections in Lorenz–Mie theory and quantum scattering: formal analogies

G. Gouesbet *

$$\sigma_{cl} = \frac{2\pi}{k^2}\sum_l (2l+1)\left(|a_l|^2 + |b_l|^2\right)$$

$$|a_l|^2 = sin^2 \alpha_l$$
$$|b_l|^2 = sin^2 \beta_l$$
$$sin^2 \delta_l = \frac{1}{2}\left(|a_l|^2 + |b_l|^2\right)$$

Classical

We then use (30)–(32) to establish

$$C^{e}_{sca} = \frac{4\pi}{k^2}\sum_{l=1}^{\infty}(2l+1)\sin^2\delta_l, \tag{41}$$

to be compared with Eq. (13) leading to the formal relation

Quantitative reletionship between quantum and classical cross-sections

$$C^{q}_{sca} = \frac{4\pi}{k^2}\sin^2\delta_0 + C^{e}_{sca}. \tag{42}$$

Therefore, to a classical electromagnetic scattering problem of LMT, we may associate a quantum scattering problem defined by

$$\left.\begin{array}{l}\delta_0 = 0\\ \sin^2\delta_l = \frac{1}{2}\left[\sin^2\alpha_l + \sin^2\beta_l\right]\end{array}\right\}. \tag{43}$$

$$\sigma_{qm} = \frac{4\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)\,sin^2\,\delta_l$$

Thermodynamic equilibrium as em scattering

Planck in "The theory of heat radiation",
pag. 34, Sect. 28

"It is thus found that, when thermodynamic equilibrium of radiation exists inside of the medium, the process of scattering produces, on the whole, no effect. The radiation falling on a volume-element from all sides and scattered from it in all directions behaves exactly as if it had passed directly through the volume-element without the least modification. Every ray loses by scattering just as much energy as it regains by the scattering of other rays."

Feynman in "The Feynman lectures on physics", Vol. 1
Sect. 41-2 "Thermal equilibrium of radiation"

"Let the gas atoms be very few and far between, so that we have an ideal oscillator with no resistance except radiation resistance. Then we consider that at thermal equilibrium the oscillator is doing two things at the same time. First, it has a mean energy kT, and we calculate how much radiation it emits. Second, this radiation should be exactly the amount that would result because of the fact that the light shining on the oscillator is scattered. Since there is nowhere else the energy can go, this effective radiation is really just scattered light from the light that is in there."

Correspondences with the blackbody

$$\sigma_{cl} = \frac{2\pi}{k^2} \sum_n (2n+1)|s_n|^2$$

Features of classical Lorenz-Mie cross section	Quantum theory of the blackbody
1) Spherical harmonics produce the dependence on $2n+1 = 2(n+1/2)$	Scaling of $n+1/2$ for the energy of the electromagnetic field
2) Fundamental spherical harmonic (monopole) causes the additional constant fraction of 1/2 in each term of the series	Fractional energy of one half the oscillator energy (zero-point energy) introduced by Planck in his second theory of the blackbody (1912) to remove inconsistencies of the first theory (1900)
3) Exponential decay approximates very well the coefficient in case of small scatterers	Boltzmann probability of a thermal oscillator excited to the *n*-th excited level
4) Sum $s = \sum_n |s_n|^2$ transforms the series of σ_{cl} into an ensemble average if the term $P_n = |s_n|^2 / s$ is valued as probability	Statistical average used to calculate the Planck thermal excitation function or the mean photon number (Bose-Einstein statistics)

The main argument

Idea:

the cross-section relates to the energy ($\sigma = W_{scatt} / I_{inc}$), then we might expect an analogy between classical and quantum em energies of the field contained in a blackbody.

(1) spherical symmetry

(2) isotropy (independence from the polarization of the em wave)

(3) homogeneous medium (permittivity independent from spatial position)

(4) non-dispersive medium (constant permittivity ε_0)

(5) non-magnetic medium (vacuum permeability μ_0)

These are the hypotheses of scalar theory of electromagnetic wave propagation and we can neglect the vectorial approach (Goodman, "Introduction to Fourier Optics", Born & Wolf, "Principles of Optics")

Scalar theory

Energy

$$\mathcal{E} = \frac{1}{2}\varepsilon_0 \int d\mathbf{r} |\mathbf{E}(\mathbf{r})|^2 + \frac{1}{2\mu_0}\int d\mathbf{r} |\mathbf{B}(\mathbf{r})|^2 \longrightarrow \mathcal{E} = \varepsilon_0 \int d\mathbf{r} |\mathbf{E}(\mathbf{r})|^2$$

Energy of the em field

In a source-free region

Field and its scalar components

$$\mathbf{E}(\mathbf{r}) = \sum_{\mathbf{k}} \mathbf{E}_{\mathbf{k}}(\mathbf{r}) = \sum_{\mathbf{k},s} E_{\mathbf{k},s}(\mathbf{r})\,\mathbf{e}_{\mathbf{k},s}$$

$$\nabla^2 E_{\mathbf{k},s}(\mathbf{r}) + k^2 E_{\mathbf{k},s}(\mathbf{r}) = 0$$ Helmholtz equation

Solution of the Helmholtz equation

$$E_{\mathbf{k},s}(\mathbf{r}) = \sum_{n=0}^{\infty} A_n\, j_n(kr) \sum_{m=-n}^{n} e^{i\alpha_{n,m}}\, Y_n^m(\vartheta,\varphi)$$

Energy of scalar components

$$\varepsilon_0 \int d\mathbf{r} |E_{\mathbf{k},s}(\mathbf{r})|^2 = \varepsilon_0 \sum_{n=0}^{\infty} (2n+1)|A_n|^2 R_n$$

$$R_n = \int_0^R dr\, r^2\, j_n^2(kr) \longrightarrow R_n = R/(2k^2) \qquad (kR >> 1)$$

$$\varepsilon_0 \int d\mathbf{r} |E_{\mathbf{k},s}(\mathbf{r})|^2 = \varepsilon_0 \frac{R}{2k^2} \sum_{n=0}^{\infty} (2n+1)|A_n|^2$$

Cross section

$$\sigma \approx \frac{1}{k^2} \sum_{n} (2n+1)|s_n|^2$$

Results

Energy proportional to the frequency

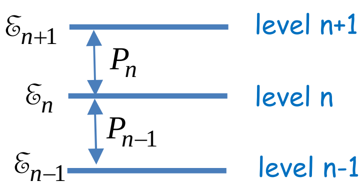
$$\sum_{\mathbf{k},s} = 2\sum_{\mathbf{k}} \rightarrow \frac{V}{\pi^2}\int k^2 dk \qquad \mathcal{E} = \eta\omega\sum_{n=0}^{\infty}|\alpha_n|^2\left(n+\frac{1}{2}\right) \qquad \eta = \frac{\varepsilon_0}{\pi^2 c}RV|E_0|^2 \qquad A_n = \alpha_n E_0 \qquad \sum_{n=0}^{\infty}|\alpha_n|^2 = 1$$

Boltzmann discretization

$$P_n = |\alpha_n|^2 = \frac{e^{-\mathcal{E}_n/k_BT}}{\sum_{n=0}^{\infty} e^{-\mathcal{E}_n/k_BT}} \qquad \mathcal{E} = \sum_{n=0}^{\infty}|\alpha_n|^2\mathcal{E}_n = \sum_{n=0}^{\infty}P_n\mathcal{E}_n \qquad \mathcal{E}_n = \eta\omega\left(n+\frac{1}{2}\right)$$

Planck's classical argument based on thermodynamics (1912)

"Űber die Begründung des Gesetz der schwarzen Strahlung", Ann. d. Phys. 342, 642-656 (1912)



$P_n \longrightarrow$ Probability that the field has energy between $\mathcal{E}_n$ and $\mathcal{E}_{n+1}$

$P_n = p(1-p)^{n-1}$ with p the probability that the field remains in its state

$$\mathcal{E} = \eta\omega\sum_{n=1}^{\infty}P_n\left(n-\frac{1}{2}\right)$$

$$\frac{1}{p} = \frac{\mathcal{E}}{\eta\omega} + \frac{1}{2}$$

entropy $S = -k_B\sum_{n=1}^{\infty}P_n \log(P_n) \longrightarrow S = k_B\left[\left(\frac{\mathcal{E}}{\eta\omega}+\frac{1}{2}\right)\log\left(\frac{\mathcal{E}}{\eta\omega}+\frac{1}{2}\right)-\left(\frac{\mathcal{E}}{\eta\omega}-\frac{1}{2}\right)\log\left(\frac{\mathcal{E}}{\eta\omega}-\frac{1}{2}\right)\right] \longrightarrow \frac{\partial S}{\partial \mathcal{E}} = \frac{1}{T}$

Energy

$$\mathcal{E} = \eta\omega\left(<n>+\frac{1}{2}\right)$$

Ensemble average

$$<n> = \sum_{n=0}^{\infty}nP_n = \frac{1}{e^{\eta\omega/k_BT}-1}$$

Planck law

$$W_T(\omega) = <n>\eta\omega\rho_\omega = <n>\frac{\eta\omega^3}{\pi^2c^3} = \frac{\eta\omega^3}{\pi^2c^3}\frac{1}{e^{\eta\omega/k_BT}-1}$$

Conclusions

Equivalence between scattering cross sections of classical electrodynamics (Lorenz-Mie) and quantum theory of wave scattering suggests a close relationship between classical and quantum physics of scattering

Thanks to the interpretation of the blackbody in terms of scattering of light, it is possible to introduce the scalar electromagnetic theory for the radiation field in a source-free region (empty cavity)

The energy is found proportional to the frequency (Planck hypothesis) and, what is more, countable by means of an integer that is related to the number of spherical harmonics (appearing in the solution of the scalar Helmholtz equation)

We can finally apply the Boltzmann statistics and follow the Planck's argument based on thermodynamics (relationship between entropy and energy). The final outcome is the Planck's law of a blackbody given in dependence of a parameter that plays the role of the Planck constant

Comparison with the method of Debye potentials

Energy of scalar components

$$\varepsilon_0 \int d\mathbf{r} \left| E_{\mathbf{k},s}(\mathbf{r}) \right|^2 = \varepsilon_0 \frac{R}{2k^2} \sum_{n=0}^{\infty} (2n+1) \left| A_n \right|^2 = \frac{\varepsilon_0 \left| E_0 \right|^2 R}{2k^2} \sum_{n=0}^{\infty} (2n+1) \left| \alpha_n \right|^2$$

$$A_n = \alpha_n E_0 \qquad \sum_{n=0}^{\infty} \left| \alpha_n \right|^2 = 1$$

Energy from vectorial theory
(Panofsky & Phillips, "Classical electricity and magnetism")

grating over a sphere of large radius. The evaluation of the time average of the scattered radiation,

$$\frac{dU_s}{dt} = \frac{1}{2} \mathrm{Re} \int |\mathbf{E}_s \times \mathbf{H}_s^*| r^2 \sin\theta \, d\theta \, d\varphi = \frac{1}{2} \mathrm{Re} \int (E_\theta H_\varphi^* - E_\varphi H_\theta^*) r^2 \, d\Omega , \tag{13–76}$$

The result of performing the integration indicated in Eq. (13–76) is

$$\frac{dU_s}{dt} = \frac{\pi E_0^2}{k^2} \sqrt{\frac{\epsilon_0}{\mu_0}} \sum_{l=1}^{\infty} (2l+1)(|a_l|^2 + |b_l|^2). \tag{13–77}$$

$$dR = c\,dt$$

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

$$U_s = \frac{\pi \varepsilon_0 E_0^2 R}{k^2} \sum_{l=1}^{\infty} (2l+1) \left(|a_l|^2 + |b_l|^2 \right)$$